

# Quark Confinement in the Deconfined Phase

K. Holland and U.-J. Wiese<sup>a</sup> \*

<sup>a</sup>Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics,  
Massachusetts Institute of Technology (MIT), Cambridge, Massachusetts 02139, U.S.A.

In cylindrical volumes with  $C$ -periodic boundary conditions in the long direction, static quarks are confined even in the gluon plasma phase due to the presence of interfaces separating the three distinct high-temperature phases. An effective “string tension” is computed analytically using a dilute gas of interfaces. At  $T_c$ , the deconfined-deconfined interfaces are completely wet by the confined phase and the high-temperature “string tension” turns into the usual string tension below  $T_c$ . Finite size formulae are derived, which allow to extract interface and string tensions from the expectation value of a single Polyakov loop. A cluster algorithm is built for the 3-d three-state Potts model and an improved estimator for the Polyakov loop is constructed, based on the number of clusters wrapping around the  $C$ -periodic direction of the cylinder.

## 1. INTRODUCTION

It has long been established that the  $Z(3)$  global symmetry of the pure  $SU(3)$  gauge theory gets spontaneously broken at high temperatures [1], leading to three distinct bulk phases, separated by interfaces. In this talk, we will present an unusual quark confining mechanism due only to the presence of these interfaces in a cylinder, relating the free energy of a single static quark to these domain walls. We believe that this is also evidence that the interfaces observed in numerical simulations correspond to physical gluonic domain walls in Minkowski space-time.

## 2. INTERFACE GAS

Pure Euclidean  $SU(3)$  gauge theory at finite temperature  $T = 1/\beta$  is described by the action  $S[A_\mu] = \int_0^\beta dt \int d^3x (1/2e^2) \text{Tr} F_{\mu\nu} F_{\mu\nu}$  and is periodic in the Euclidean time direction. The Polyakov loop  $\Phi(\vec{x})$  is constructed from the Euclidean time component of the gauge field  $A_4(\vec{x}, t)$  and under gauge transformations  $g(\vec{x}, t)$  which are also periodic in Euclidean time, both the action and the Polyakov loop are unchanged. However, if the gauge transformations differ by a cen-

ter element i.e.  $g(\vec{x}, t + \beta) = g(\vec{x}, t)z, z \in Z(3)$ , the action is invariant but the Polyakov loop changes into  $\Phi'(\vec{x}) = \Phi(\vec{x})z$ . The expectation value  $\langle \Phi \rangle = \exp(-\beta F)$  measures the free energy,  $F$ , of a static quark. In the confined phase,  $F$  diverges and  $\langle \Phi \rangle$  vanishes, while in the deconfined phase  $F$  is finite and  $\langle \Phi \rangle \neq 0$ . Hence, the  $Z(3)$  center symmetry is spontaneously broken at high temperatures.

With spontaneous symmetry breaking, the spatial boundary conditions and the manner of the infinite volume limit are important. We consider a spatial volume of size  $L_x \times L_y \times L_z$ . If we apply periodic boundary conditions,  $\langle \Phi \rangle$  vanishes even in the deconfined phase, due to Gauss' law in a periodic volume [2] — topologically, we cannot have a single quark in a periodic box because its center electric flux cannot go to infinity, it must end in an anti-quark.

We apply  $C$ -periodic boundary conditions in the  $z$ -direction only [3]. When a  $C$ -periodic field is translated by  $L_z$ , it is replaced by its charge conjugate. For example, for  $C$ -periodic gluons,  $A_\mu(\vec{x} + L_z \vec{e}_z, t) = A_\mu(\vec{x}, t)^*$ , where  $*$  means complex conjugate. Physically, we can have a single quark in a volume, partnered with an anti-quark on the other side of the  $C$ -periodic boundary. Imposing  $C$ -periodicity explicitly breaks the  $Z(3)$  symmetry [4], but this symmetry breaking disappears in the infinite volume limit. With  $C$ -

\*Talk presented by K.H. This work is supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative agreement DE-FC02-94ER40818

periodic boundary conditions,  $\langle \Phi \rangle$  is always non-zero in a finite volume.

The original motivation for applying  $C$ -periodicity in the long direction of a cylinder was to extract the string tension  $\sigma$  from numerical simulations measuring  $\langle \Phi \rangle$  in the confined phase. A single static quark sits in the cylinder, connected by a tube of gluons to its anti-quark partner on the other side of the  $C$ -periodic boundary. Then we have  $\langle \Phi \rangle = \Sigma_0 \exp(-\beta \sigma L_z)$  and so the free energy of a single quark in the cylinder is

$$F = -\frac{1}{\beta} \log \Sigma_0 + \sigma L_z.$$

There are several advantages to using this method in a numerical study. Firstly, it is much easier to measure the Polyakov loop than a Wilson loop or some other correlator. Secondly, in a periodic box of size  $L$ , the Wilson loop cannot be larger than  $L/2$  in its spatial extent, whereas this technique allows us to exploit the entire volume.

We can also investigate  $\langle \Phi \rangle$  in the deconfined phase at temperatures  $T > T_c$ , where the three distinct deconfined phases coexist [5]. They are distinguished by different values of  $\langle \Phi \rangle$  — one expectation value is  $\Phi^{(1)} = (\Phi_0, 0)$ , which is rotated by  $Z(3)$  transformations to give  $\Phi^{(2)}$  and  $\Phi^{(3)}$ . A typical configuration in a cylinder consists of several bulk phases, aligned along the  $z$ -direction, separated by deconfined-deconfined interfaces. The interfaces cost free energy  $F$  proportional to their area  $A = L_x L_y$ , such that the interface tension is given by  $\alpha_{dd} = F/A$ . The expectation value of the Polyakov loop can be calculated from a dilute gas of interfaces [6]. The interface expansion of the partition function can be viewed as

$$Z = \boxed{d_1} + \boxed{d_2} \boxed{d_3} + \boxed{d_3} \boxed{d_2} + \dots$$

The first term has no interfaces and thus the whole cylinder is filled with deconfined phase  $d_1$  only. An entire volume filled with either phase  $d_2$  or  $d_3$  would not satisfy the boundary conditions. The second and third terms have one interface separating phases  $d_2$  and  $d_3$ . Here,  $C$ -periodic boundary conditions exclude phase  $d_1$ . For each

configuration, we integrate the Boltzmann weight over all possible locations of the interface, then we sum the Boltzmann weights for all configurations with all possible numbers of interfaces. Summing the interface expansion to all orders, we obtain  $Z = \exp(-\beta f_d A L_z + 2\gamma \exp(-\beta \alpha_{dd} A) L_z)$ . Here,  $f_d$  is the bulk deconfined free energy density and  $\gamma$  is a factor resulting from capillary wave fluctuations of the interfaces. Note that in three dimensions,  $\gamma$  is to leading order independent of the area  $A$  [7]. To calculate  $\langle \Phi \rangle$ , we simply include the Polyakov loop expectation value of each configuration with the Boltzmann weight in the interface expansion. Summing to all orders, we obtain  $\langle \Phi \rangle = \Phi_0 \exp(-3\gamma \exp(-\beta \alpha_{dd} A) L_z)$ , from which we calculate that the free energy of a single static quark in a  $C$ -periodic cylinder is given by

$$F = -\frac{1}{\beta} \log \Phi_0 + \frac{3\gamma}{\beta} \exp(-\beta \alpha_{dd} A) L_z.$$

This result is counter intuitive. Although we are in the deconfined phase, the quark's free energy diverges in the limit  $L_z \rightarrow \infty$ , as long as the cross section  $A$  of the cylinder remains fixed. This is the behavior one typically associates with confinement. In fact,  $\sigma' = (3\gamma/\beta) \exp(-\beta \alpha_{dd} A)$  plays the role of the “string tension”, even though there is no physical string connecting the quark to its anti-quark partner on the other side of  $C$ -periodic boundary. Because the deconfined-deconfined interfaces can lead to the divergence of a quark's free energy and have physically observable consequences, we believe that they are more than just Euclidean field configurations.

Because the phase transition is of first order [8], as we approach  $T_c$ , the confined phase can coexist with the three deconfined phases, so we can also have confined-deconfined interfaces with an interface tension  $\alpha_{cd}$ . At  $T_c$ , there are two possibilities [9,10]. If we have  $\alpha_{dd} = 2\alpha_{cd}$ , a deconfined-deconfined interface always splits into two confined-deconfined interfaces, separated by a film of confined phase — this is called complete wetting. If  $\alpha_{dd} < 2\alpha_{cd}$ , both deconfined-deconfined and confined-deconfined interfaces are stable — this is called incomplete wetting. Numerical simulations indicate that the gluon system undergoes complete wetting [11,6]. In that

case, the interface expansion of the partition function is

$$Z = \boxed{c} + \boxed{d_1} + \sum_i \left\{ \boxed{c} \boxed{d_i} \boxed{c} + \boxed{d_i} \boxed{c} \boxed{d_i^*} \right\} + \dots$$

The sum over  $i$  extends over the three deconfined phases and  $d_i^*$  denotes the charge-conjugate of  $d_i$ . For example,  $d_1^* = d_1$  and  $d_2^* = d_3$ . Note that due to complete wetting, one always has an even number of interfaces. As before, we sum the expansion to all orders to calculate the partition function and  $\langle \Phi \rangle$ , obtaining

$$\langle \Phi \rangle = \frac{\Phi_0 \exp(\beta x L_z) + \Sigma_0 \exp(-\beta(x + \sigma)L_z)}{2 \cosh[\beta L_z \sqrt{x^2 + (3\delta^2/\beta^2) \exp(-2\beta\alpha_{cd}A)}]}.$$

Here,  $\delta$  is the factor characterizing capillary wave fluctuations of the confined-deconfined interfaces and  $x = \frac{1}{2}(f_c - f_d)A$  measures the bulk free energy difference between confined and deconfined phases. From this expression, we can extract an effective “string tension”  $\sigma'$  which, as we lower the temperature further into the confined regime, reduces to the standard string tension  $\sigma$  in the large  $A$  limit, as expected.

We can also consider the incomplete wetting scenario by including configurations with deconfined-deconfined interfaces at  $T \approx T_c$ . The effective “string tension” we extract again matches  $\sigma$  as we move into the confined regime.

### 3. IMPROVED ESTIMATOR

Using the analytically derived finite size formulae, we can determine the interface tensions  $\alpha_{dd}$  and  $\alpha_{cd}$  from numerical simulations. Because simulating  $SU(3)$  gauge theory is computationally intensive, we select a simpler model, the 3-d three-state Potts model. Its action is given by  $S = -\beta \sum_{x,\mu} \delta_{\Phi_x, \Phi_{x+\hat{\mu}}}$ , where  $\Phi_x \in Z(3)$ . This model has a  $Z(3)$  symmetry and a first order phase transition between three distinct ordered (deconfined) phases and one disordered (confined) phase. Here,  $\langle \Phi \rangle$  corresponds to the average spin. We have developed a single cluster algorithm with an improved estimator for measuring  $\langle \Phi \rangle$  in a cylinder with  $C$ -periodicity im-

posed in the long direction. As we grow the cluster, we keep account of how many times the cluster crosses the  $C$ -periodic boundary. If the cluster wraps around the cylinder in the long direction an odd number of times, we count a 1. Otherwise, we have a non-wrapping cluster and we count a 0. This is because a non-wrapping cluster is not aware of the  $C$ -periodicity and so can flip to any of the three deconfined phases, making on average a zero contribution to  $\langle \Phi \rangle$ , whereas a wrapping cluster can only be in deconfined phase  $d_1$  satisfying  $C$ -periodicity. Because we have an improved estimator, we can accurately determine very small quantities. Work on simulations to extract the interface tensions is in progress.

### REFERENCES

1. L.D. McLerran and B. Svetitsky, Phys. Rev. D24 (1981) 450.
2. E. Hilf and L. Polley, Phys. Lett. B131 (1983) 412.
3. A.S. Kronfeld and U.-J. Wiese, Nucl. Phys. B357 (1991) 521.
4. U.-J. Wiese, Nucl. Phys. B375 (1992) 45.
5. K. Holland and U.-J. Wiese, hep-lat/9702014.
6. B. Grossmann, M.L. Laursen, T. Trappenberg and U.-J. Wiese, Nucl. Phys. B396 (1993) 584.
7. V. Privman and M.E. Fisher, J. Stat. Phys. 33 (1983) 385; E. Brezin and J. Zinn-Justin, Nucl. Phys. B257 [FS14] (1985) 867.
8. R. V. Gavai, F. Karsch and B. Petersson, Nucl. Phys. B322 (1989) 738; M. Fukugita, M. Okawa and U. Ukawa, Phys. Rev. Lett. 63 (1989) 1768.
9. Z. Frei and A. Patkós, Phys. Lett. B229 (1989) 102.
10. T. Trappenberg and U.-J. Wiese, Nucl. Phys. B372 (1992) 703.
11. K. Kajantie, L. Kärkkäinen and K. Rummukainen, Nucl. Phys. B333 (1990) 100; Nucl. Phys. B357 (1991) 693.